Progressive Mathematics Initiative® (PMI®)
MATH6433: Learning and Teaching Geometry

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Course Credit: 4.0 NJCTL credits

Dates & Times:
This is a 4-credit, self-paced course, covering 14 modules of content. The exact number of hours that you can expect to spend on each module will vary based upon the module coursework, as well as your study style and preferences. You should plan to spend approximately 15 hours per credit working online, and up to 30 hours per credit working offline.

Graduate Student Handbook: www.njctl.org/graduate-handbook/

COURSE DESCRIPTION:
This course is for teachers who intend to become middle school and/or high school mathematics teachers. This is a proof-based course that requires teachers to develop skills in writing deductive geometric proofs. It provides teachers with the background knowledge, at an introductory college level, of Euclidean Geometry and applies that knowledge when writing proofs and problem solving. While the focus will be on two-column proofs, there will be some exposure to other types of proof.

This course includes topics taken from the middle school and high school Common Core State Standards for Mathematics (CCSS) and the New Jersey State Standards for Mathematics. The focus is the conceptual development of points, lines, planes, angles & proofs and their application to the understanding of triangles, similar triangles & trigonometry, congruent triangles, circles, analytic geometry, transformations, quadrilaterals, area, volume, and probability.

STUDENT LEARNING OUTCOMES:
Upon completion of the course, the student will be able to:
1. Demonstrate an understanding of mathematics concepts of Geometry, including how to teach them, detailed in the module learning outcomes below.
2. Integrate PMI materials (including presentations, labs, practice problems, etc.) to support student learning and deliver effective instruction.
3. Create a social constructivist learning environment through the use of formative assessment questions, interpreting the results of this assessment to effectively facilitate student-led discussions that support deeper understanding of the content.
4. Integrate multiple attempts to demonstrate student mastery of content knowledge, as encouraged/fostered by the PMI pedagogy.
5. Implement learning plans that are aligned to Common Core standards and allow for differentiation based on the needs of learners.

TEXTS, READINGS, INSTRUCTIONAL RESOURCES:
Required Texts:
- This course uses a free digital textbook accessible at:
  https://njctl.org/courses/math/geometry/
- Participants will download SMART Notebook presentations, homework files, labs, and teacher resources from the PMI Geometry course

Recommended Readings:
- Related articles within discussion prompts

COURSE REQUIREMENTS:
In order to receive a Passing grade, the participant must complete the following course requirements:
1. Activities: A number of different learning activities will ensure participant engagement and learning in the course. These include:
   - Engage in video module lessons which demonstrate minimized direct instruction followed by frequent formative assessment.
   - Completion of formative assessments aligned to learning objectives which include detailed analysis when answered incorrectly.
   - Interaction with module discussion boards that allow conversation with peers and course instructors about the module's content, delivering that content to students. Discussion boards also serve as a place to ask and answer questions related to the module's content.
2. Short Answer Assignment: Each module requires one (1) original response to a given prompt. These prompts are typically based upon course lessons and require teachers to analyze, reflect, and make connections between the module's content and their own classroom practice.
3. Mastery Exercises: For each module, these multiple-choice question quizzes assess the content knowledge gained in a module.   Participants have the opportunity to retake; random questions are pulled from a larger question bank on each attempt ensuring varied questions.

4. Virtual Labs: In each module, a virtual lab write-up will be submitted. Virtual labs are interactive lab simulations that promote discovery-based student learning through real-world applications and analysis.
5. Module Exam: One is completed at the end of each module. It is a culminating exam consisting of multiple choice and free response questions aligned to the standards and objectives of the module.
6. Reflection Paper: At the end of the course, participants are required to reflect on the knowledge taught in the course, make connections, and compare/contrast their current pedagogy with new strategies gained in this assignment.
7. Final Exam: At the end of the course, a comprehensive exam consisting of Multiple Choice and Free Response questions assesses the content knowledge learned throughout the course.

GRADE DISTRIBUTION AND SCALE:

Grade Distribution:

Module Exams	70%
Final Exam	10%
Labs	6%
Short Answer Assignments	6%
Mastery Exercises	6%
Reflection Paper	2%

Grade Scale:

A	93 – 100
A-	90 – 92
B+	86 – 89
B	83 – 86
B-	80 – 82
C+	77 – 79
C	73 – 76
C-	70 – 72
D	60.0 – 69.9
F	59.9 or below

GRADING RUBRIC:
The following rubric is used to score:
- Short Answer Assignment – 6% of grade
- Reflection Paper – 2% of grade

The minimum possible score for this rubric is 4 points, and the score will be converted to the minimum grade available in this module (which is zero unless the scale is used). The maximum

score of 25 points will be converted to the maximum grade.

Intermediate scores will be converted respectively and rounded to the nearest available grade. If a scale is used instead of a grade, the score will be converted to the scale elements as if they were consecutive integers.

	Meets Expectation	**Approaches Expectation**	**Below Expectation**	**Limited Evidence**
	7 points	*5 points*	*3 points*	*1 point*
Content	• Demonstrates excellent knowledge of concepts, skills, and theories relevant to topic.	• Demonstrates fair knowledge of concepts, skills, and theories.	• Demonstrates incomplete or insubstantial knowledge of concepts, skills, and theories.	• Demonstrates little or no knowledge of concepts, skills, and theories.
Depth of Reflection	• Content is well supported and addresses all required components of the assignment.	• Content is partially supported; addresses most of the required components of the assignment.	• Content contains major deficiencies; addresses some of the required components of the assignment.	• Content is not supported and/or includes few of the required components of the assignment.
Evidence and Practice	• Response shows strong evidence of synthesis of ideas presented and insights gained throughout the entire course. The implications of these insights for the respondent's overall teaching practice are thoroughly detailed, as applicable.	• Writing is mostly clear, concise, and well organized with good sentence/paragraph construction. Thoughts are expressed in a coherent and logical manner. There are no more than five spelling, grammar, or syntax errors per page of writing.	• Response is missing some components and/or does not fully meet the requirements indicated in the instructions. Some questions or parts of the assignment are not addressed. Some attachments and additional documents, if required, are missing or unsuitable for the purpose of the assignment.	• Response excludes essential components and/or does not address the requirements indicated in the instructions. Many parts of the assignment are addressed minimally, inadequately, and/or not at all.
	4 points	*3 points*	*2 points*	*1 point*
Writing Quality	• Writing is well-organized, clear, concise, and focused; no errors.	• Some minor errors or omissions in writing organization, focus, and clarity.	• Some significant errors or omissions in writing organization, focus, and clarity.	• Numerous errors in writing organization, focus, and/or clarity.

The following rubric is used to score:
- Labs – 6% of grade

The minimum possible score for this rubric is 2 points, and the score will be converted to the minimum grade available in this module (which is zero unless the scale is used). The maximum score of 14 points will be converted to the maximum grade.

Intermediate scores will be converted respectively and rounded to the nearest available grade. If a scale is used instead of a grade, the score will be converted to the scale elements as if they were consecutive integers.

	Meets Expectation	Approaches Expectation	Below Expectation	Limited Evidence
	7 points	*5 points*	*3 points*	*1 point*
Completeness	• Lab write-up is complete with no missing fields.	• Lab write-up has 1-2 missing fields.	• Lab write up has 3-5 missing fields.	• There are more than 5 missing fields on the lab write-up.
Calculations	• All answers are calculated correctly.	• Most answers are calculated correctly, but there are 1-2 minor calculation errors.	• Most answers are calculated correctly, but there are multiple minor calculation errors, or 1-2 gross miscalculations.	• There are calculation errors throughout the lab.

The remaining types of assignments are not scored using a rubric. These assignments are scored using percentage correct to assign a letter grade. The assignments in this manner are as follows:
- Mastery Exercises – 6% of grade
- Module Exams – 70% of grade
- Final Exam – 10% of grade

Mastery Exercises can be retaken as many times as desired to ensure a high score. Due to the nature of these assignments, each time they are taken, they will be composed of unique questions pulled randomly from a larger question bank.

Module and Final Exams are scored using a curve, which allows us to keep content exams rigorous. Module Exams can be retaken one time. Final Exams cannot be retaken.

ACADEMIC STANDING:
NJCTL has established standards for academic good standing within a student's academic program. Students enrolled in any NJCTL online course must receive an 80 or higher to successfully complete a course and receive credit for that course. An 80 is equivalent to a GPA

of 2.7 or B-. Additionally, students in an endorsement program must receive a cumulative GPA of 3.0 for all courses combined in order to successfully complete the program.

ACADEMIC INTEGRITY:
Students must assume responsibility for maintaining honesty in all work submitted for credit and in any other work designated by the instructor of the course. Academic dishonesty includes cheating, fabrication, facilitating academic dishonesty, plagiarism, reusing /repurposing your own work, unauthorized possession of academic materials, and unauthorized collaboration.

CITING SOURCES WITH APA STYLE:
All students are expected to follow proper writing and APA requirements when citing in APA (based on the APA Style Manual, 6th edition) for all assignments.

DISABILITY SERVICES STATEMENT:
We are committed to providing reasonable accommodations for all persons with disabilities. Any student with a documented disability requesting academic accommodations should contact the Dean of Students, Melissa Axelsson, for additional information to coordinate reasonable accommodations for students with documented disabilities (melissa@njctl.org).

NETIQUETTE:
Respect the diversity of opinions among the instructor and classmates and engage with them in a courteous, respectful, and professional manner. All posts and classroom communication must be conducted in accordance with the student code of conduct. Think before you push the Send button. Did you say just what you meant? How will the person on the other end read the words?

Maintain an environment free of harassment, stalking, threats, abuse, insults or humiliation toward the instructor and classmates. This includes, but is not limited to, demeaning written or oral comments of an ethnic, religious, age, disability, sexist (or sexual orientation), or racist nature; and the unwanted sexual advances or intimidations by email, or on discussion boards and other postings within or connected to the online classroom.

If you have concerns about something that has been said, please let your instructor know.

CLASS SCHEDULE:

Module	Module Learning Outcomes	Assignments
1 – Points, Lines, & Planes	- Describe and identify points, lines, and planes. - Determine whether segments are congruent. - Apply the Ruler Postulate and the Segment Addition Postulate to calculate the length of a line segment. - Apply the Segment Addition Postulate and the rules of algebra to write and solve algebraic equations to find the value of a variable. - Construct congruent segments using a compass and straightedge, and/or dynamic geometry software. - Construct arguments to prove theorems about lines.	- Short Answer - Lab - Mastery Exercises - Module Exam

Module	Objectives	Assessments
2 – Angles & Introduction to Proof	- Identify and classify angles based on its measurement and name them. - Apply the Angle Addition Postulate to calculate the measurement of angles and angle pair relationships. - Measure angles using a protractor. - Construct an angle congruent to any given angle using a compass and straightedge, and/or dynamic geometry software. - Construct arguments to prove theorems about angles. - Apply their knowledge of angle bisectors to calculate the measurements of angles.	- Short Answer - Lab - Mastery Exercises - Module Exam
3 – Parallel Lines	- Identify pairs of angles formed by two lines cut by a transversal. - Apply the properties of angles formed by parallel lines and a transversal. - Use information about angles to conclude whether or not lines are parallel/perpendicular. - Identify corresponding, alternate-interior, alternate-exterior, and same-side interior angles and then find the missing angles or solve for x. - Construct parallel lines.	- Short Answer - Lab - Mastery Exercises - Module Exam
4 – Triangles	- Identify triangles by sides and angles. - Write and solve algebraic equations to find the missing angle measurement, and/or the value of the variable using the Triangle Sum Theorem and Exterior Angles Theorem. - Identify which side of a triangle is the largest, knowing angle measure. - Identify which angle is the largest, knowing side lengths. - Write and solve proportions to find the missing side lengths in similar triangles. - Determine whether or not triangles are similar based on the given information. - Construct arguments and/or reasons to prove that triangles are similar.	- Short Answer - Lab - Mastery Exercises - Module Exam
5 – Similar Triangles & Trigonometry	- Similarity ratios to solve problems. - Use trigonometric ratios to solve right triangles. - Derive the trigonometry formulas. - Solve problems and identify right triangles using Pythagorean Theorem. - Derive the 45-45-90 & 30-60-90 triangle theorems. - Solve problems using the 45-45-90 & 30-60-90 triangle theorems.	- Short Answer - Lab - Mastery Exercises - Module Exam
6 – Congruent Triangles	- Show triangles are congruent and solve problems based on that congruence. - State the congruence of corresponding parts knowing congruent triangles. - Complete and write proofs of triangles. - Find the congruent angles of an isosceles triangle given congruent sides. - Find the congruent sides of an isosceles triangle given congruent angles.	- Short Answer - Lab - Mastery Exercises - Module Exam

7 – Circles	- Identify the parts of a circle. - Find the measure of angles given angles with vertices at the center, inside the circle, on the circle, or outside the circle. - Calculate the length of an arc in a circle. - Calculate the measurement of angles and arcs in radians. - Calculate the lengths of tangents, secant segments, and chords depending on the location of the intersection.	- Short Answer - Lab - Mastery Exercises - Module Exam
8 – Analytic Geometry	- Calculate the distance between 2 points using the distance formula. - Calculate the midpoint of a segment when given 2 points. - Calculate the missing endpoint of a segment when given the midpoint and one endpoint of a segment. - Calculate the missing point(s) of a segment that partition a segment into 2 segments with a given ratio. - Determine the slope of a line that is parallel/perpendicular to a given line. - Write the equation of a line that is parallel/perpendicular to a given line passing through another point in the coordinate plane. - Prove that 2 triangles are congruent in a coordinate plane using the distance, midpoint, and/or slope formulas in combination with the 5 methods of proving triangles are congruent. - Use the distance, midpoint, and/or slope formula to classify a triangle based on the coordinates of its vertices. - Find the center, radius, and points on a circle given the equation of the circle. - Calculate the value of a missing variable in the equation of a circle using the algebraic strategy of "Completing the Square".	- Short Answer - Lab - Mastery Exercises - Module Exam
9 – Transformations	- Identify rigid motions and images. - Describe transformations in the coordinate plane using function notation. - Translate, reflect, rotate and dilate a figure using a variety of tools. - Identify line or rotational symmetry of a figure, if it exists. - Construct an equilateral triangle, square and regular hexagon using a variety of methods (compass & straightedge, string, and/or dynamic geometric software). - Given congruent figures or similar figures, find a sequence of transformations that maps one figure onto the other.	- Short Answer - Lab - Mastery Exercises - Module Exam
10 – Quadrilaterals	- Find the sum of interior and exterior angles in polygons. - Find the measures of interior and exterior angles in polygons. - Use the properties of quadrilaterals to find the measurements of angles and the lengths of sides. - Use the properties of quadrilaterals to write and solve algebraic equations. - Identify the members of the quadrilaterals family. - Construct quadrilaterals by applying sufficient conditions. - Construct arguments to write proofs involving quadrilaterals.	- Short Answer - Lab - Mastery Exercises - Module Exam

11 – Area of Figures	- Calculate the area of a rectangle when side lengths are unknown using other formulas (e.g. Pythagorean Theorem). - Calculate the area of any triangle using trigonometric ratios after drawing an auxiliary line from a vertex perpendicular to the opposite side. - Prove the Law of Sines formula and use it to solve problems. - Calculate the area of any parallelogram using trigonometric ratios and its area formula. - Calculate the area of any regular polygon using trigonometric ratios and its area formula. - Derive and use the area of a circle formula from the area formula for any regular polygon. - Use the area of a circle formula and the relationship that exists between the degrees of the arc and the area of its sector to derive the equation used to calculate the area of a sector. - Derive the formula for the area of a trapezoid and use it to solve problems. - Derive the formula for the area of a rhombus and use it to solve problems. - Derive the formula for the area of a kite and use it to solve problems. - Calculate the area of complex figures using the area formulas of 2-D shapes. - Apply the area formulas and the distance formula to solve real-world problems.	- Short Answer - Lab - Mastery Exercises - Module Exam
12 – 3D Geometry	- Identify the vertices, edges and faces in a polyhedron. - Calculate the number of vertices, edges, or faces in a polyhedron using Euler's Theorem. - Identify the cross-sectional shapes formed by a solid and an intersecting 2-dimensional plane. - Use orthographic projections of a solid to draw a 3-dimensional representation of the solid and vice-versa. - Calculate the lateral area, surface area, and volume for prisms, pyramids, cylinders and cones. - Calculate the slant height of a regular pyramid and right cone. - Calculate surface area and volume of a sphere. - Calculate the circumference and area of a great circle of a sphere. - Compare volumes of surfaces using Cavalieri's Principle. - Calculate the similarity ratio of similar figures. - Use the concepts of lateral area, surface area, and/or volume to solve real-world application problems.	- Short Answer - Lab - Mastery Exercises - Module Exam

13 – Probability	- Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. - Use theoretical probability to determine the likelihood of an event occurring. - Represent probabilities in multiple ways, such as fractions, decimals, percentages, and geometric area models. - Determine the probability of an event using number lines, coordinate planes and formulas for geometric figures. - Determine the probability of an event involving area.	- Short Answer - Lab - Mastery Exercises - Module Exam
14 – Reflection & Final Exam	- Review topics from course - Zoom meeting with course instructor, as needed	- Reflection Paper - Final Exam