NEW JERSEY CENTER
FOR TEACHING & LEARNING

Progressive Mathematics Initiative® (PMI®)
MATH6467: High School Mathematics Capstone & Praxis Preparation

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Course Credit: 3.0 NJCTL credits

Dates & Times:
This is a 3-credit, self-paced course, covering 12 modules of content. The exact number of hours that you can expect to spend on each module will vary based upon the module coursework, as well as your study style and preferences. You should plan to spend approximately 15 hours per credit working online, and up to 30 hours per credit working offline.

Graduate Student Handbook: www.njctl.org/graduate-handbook/

COURSE DESCRIPTION:
This course is for teachers to review and extend their prior study of mathematics in the realms of number & quantity; algebra; functions; calculus; geometry; probability & statistics; and discrete mathematics. This capstone course also serves as a review for the Mathematics Praxis Test (5165).

STUDENT LEARNING OUTCOMES:
Upon completion of the course, the student will be able to:

1. Understand and work with mathematical concepts, to reason mathematically, to make conjectures, see patterns, and to justify statements using informal, logical arguments.
2. Demonstrate the ability to solve problems by integrating knowledge of different areas of mathematics.
3. Implement the use of various representations of concepts.
4. Solve problems that have several solution paths.
5. Effectively demonstrate the use of technology in mathematics education.

6. Develop mathematical models and use them to solve real-world problems.
7. Construct viable arguments to write simple proofs.

TEXTS, READINGS, INSTRUCTIONAL RESOURCES:
Required Texts:

- HS Mathematics Praxis Preparation uses a free, digital textbook that is available within the course modules as PDFs. Readings are derived from the secondary math courses located at: https://njctl.org/courses/math/

Recommended Readings:

- Related articles within short answer prompts

COURSE REQUIREMENTS:
In order to receive a Passing grade, the participant must complete the following course requirements:

1. Activities: A number of different learning activities will ensure participant engagement and learning in the course. These include:
    - Engage in video module lessons which demonstrate minimized direct instruction followed by frequent formative assessment.
    - Completion of formative assessments aligned to learning objectives which include detailed analysis when answered incorrectly.
    - Interaction with module discussion boards that allow conversation with peers and course instructors about the module's content, delivering that content to students. Discussion boards also serve as a place to ask and answer questions related to the module's content.
2. Mastery Exercises: For each module, these multiple-choice question quizzes assess the content knowledge gained in a module. Participants have the opportunity to retake; random questions are pulled from a larger question bank on each attempt ensuring varied questions.
3. Virtual Labs: Virtual labs are interactive lab simulations that are aligned to further prepare participants for the Praxis exam covering topics about the On-Screen Calculator, as well as Geometric Constructions.
4. Module Exam: One is completed at the end of each module. It is a culminating exam consisting of praxis-like multiple-choice questions aligned to the exam objectives.
5. Reflection Paper: At the end of the course, participants are required to reflect on the knowledge taught in the course, make connections, and compare/contrast their current pedagogy with new strategies gained in this assignment.
6. Final Exam: At the end of the course, a comprehensive exam consisting of Multiple-Choice questions assesses the content knowledge learned throughout the course in preparation for the Praxis exam.

GRADE DISTRIBUTION AND SCALE:

Grade Distribution:

Module Exams	70%
Final Exam	20%
Mastery Exercises	6%
Labs	2%
Reflection Paper	2%

Grade Scale:

A	93 – 100
A-	90 – 92
B+	86 – 89
B	83 – 86
B-	80 – 82
C+	77 – 79
C	73 – 76
C-	70 – 72
D	60.0 – 69.9
F	59.9 or below

ACADEMIC STANDING:
NJCTL has established standards for academic good standing within a student's academic program. Students enrolled in any NJCTL online course must receive an 80 or higher to successfully complete a course and receive credit for that course. An 80 is equivalent to a GPA of 2.7 or B-. Additionally, students in an endorsement program must receive a cumulative GPA of 3.0 for all courses combined in order to successfully complete the program.

ACADEMIC INTEGRITY:
Students must assume responsibility for maintaining honesty in all work submitted for credit and in any other work designated by the instructor of the course. Academic dishonesty includes cheating, fabrication, facilitating academic dishonesty, plagiarism, reusing /repurposing your own work, unauthorized possession of academic materials, and unauthorized collaboration.

CITING SOURCES WITH APA STYLE:
All students are expected to follow proper writing and APA requirements when citing in APA (based on the APA Style Manual, 6th edition) for all assignments.

DISABILITY SERVICES STATEMENT:
We are committed to providing reasonable accommodations for all persons with disabilities. Any student with a documented disability requesting academic accommodations should contact the Dean of Students, Melissa Axelsson, for additional information to coordinate reasonable

accommodations for students with documented disabilities (melissa@njctl.org).

NETIQUETTE:
Respect the diversity of opinions among the instructor and classmates and engage with them in a courteous, respectful, and professional manner. All posts and classroom communication must be conducted in accordance with the student code of conduct. Think before you push the Send button. Did you say just what you meant? How will the person on the other end read the words?

Maintain an environment free of harassment, stalking, threats, abuse, insults or humiliation toward the instructor and classmates. This includes, but is not limited to, demeaning written or oral comments of an ethnic, religious, age, disability, sexist (or sexual orientation), or racist nature; and the unwanted sexual advances or intimidations by email, or on discussion boards and other postings within or connected to the online classroom.

If you have concerns about something that has been said, please let your instructor know.

CLASS SCHEDULE:

Module	Module Learning Outcomes	Assignments
1 – Numbers and Quantity	● Understand the properties of exponents. ● Understand the properties of rational and irrational numbers, and the interactions between those sets of numbers. ● Know how to solve problems by reasoning quantitatively (e.g. dimensional analysis, reasonableness of solutions). ● Understand the structure of the natural, integer, rational, real and complex number systems and how the basic operations (+,-,x, and ÷) on numbers in these systems are performed. ● Understand how to work with complex numbers when solving polynomial equations and rewriting polynomial expressions. ● Understand how to perform operations on matrices and how to use matrices in applications. ● Understand how to solve problems involving ratios, proportions, averages, percents and metric and traditional unit conversions. ● Know how to analyze both precision and accuracy in measurement situations. ● Understand various ways to represent and compare very large and very small numbers (e.g. scientific notation, orders of magnitude). ● Understand how to both estimate and perform calculations on very large and very small quantities.	● Mastery Exercises ● Module Exam

2 - Algebra	● Understand how to write algebraic expressions in equivalent form. ● Understand how to perform arithmetic operations on polynomials. ● Understand the relationship between zeros of polynomial functions (including their graphical representation) and factors of the related polynomial expressions. ● Understand how to use polynomial identities (e.g., difference of square, sum and difference of cubes) to solve problems. ● Understand how to rewrite rational expressions and perform arithmetic operations on rational expressions. ● Understand how to create equations and inequalities that describe relationships. ● Understand how to justify the reasoning process used to solve equations, including analysis of potential extraneous solutions. ● Understand how varied techniques (e.g., graphical, algebraic) are used to solve equations and inequalities in one variable. ● Understand how varied techniques (e.g., graphical, algebraic, matrix) are used to solve systems of equations and inequalities. ● Understand the properties of number systems under various operations. ● Understand the concept of rate of change of nonlinear functions. ● Understand the concepts of intercept(s) of a line and slope as a rate of change. ● Understand how to find the zero(s) (real and complex) of functions.	● Mastery Exercises ● Module Exam

3 - Functions	● Understand the function concept and the use of function notation. ● Understand how to find the domain and range of a function and a relation. ● Understand how function behavior is analyzed using different representations (e.g., graphs, mappings, tables). ● Understand how functions and relations are used to model relationships between quantities. ● Understand how new functions are obtained from existing functions (e.g., compositions, transformations, inverses). ● Understand differences between linear, quadratic, and exponential models, including how their equations are created and used to solve problems. ● Understand how to construct the unit circle and how to use it to find values of trigonometric functions for all angle measures in their domains. ● Understand how periodic phenomena are modeled using trigonometric functions. ● Understand the application of trigonometric identities (e.g., Pythagorean, double angle, half angle, sum of angles, difference of angles). ● Know how to interpret representation of functions of two variables (e.g., three-dimensional graphs, tables). ● Understand how to solve equations (e.g., trigonometric, logarithmic, exponential).	● Mastery Exercises ● Module Exam

4 - Geometry	• Understand transformations in a plane. • Understand how to prove geometric theorems, such as those about lines and angles, triangles, and parallelograms. • Understand how geometric constructions are made with a variety of tools and methods. • Understand congruence and similarity in terms of transformations. • Understand how trigonometric ratios are defined in right triangles. • Understand how trigonometry is applied to general triangles. • Understand and apply theorems about circles. • Understand arc length and area measurements of sectors of circles. • Know how to translate between a geometric description (eg.g, focus, asymptotes, directrix and an equation for a conic section). • Understand how to use coordinate geometry to algebraically prove simple geometric theorems. • Understand how perimeter, area, surface area, and volume formulas are used to solve problems. • Know how to visualize relationships (e.g., cross section, nets, rotations) between two-dimensional and three-dimensional objects. • Know how to apply geometric concepts in real-world situations. • Understand the properties of parallel and perpendicular lines, triangles, quadrilaterals, polygons, and circles and how they can be used in problem solving.	• Lab • Mastery Exercises • Module Exam
5 – Probability & Statistics	• Know how to summarize, represent, and interpret data collected from measurements on a single variable (e.g., box plots, dot plots, normal distributions). • Understand how to summarize, represent, and interpret data collected from measurements on two variables, either categorical or quantitative (e.g., scatterplots, time series). • Understand how to create and interpret linear regression models (e.g., rate of change, intercepts, correlation coefficient). • Understand statistical processes and how to evaluate them. • Understand how to make inferences and justify conclusions from samples, experiments, and observational studies. • Understand the concepts of independence and conditional probability and how to apply these concepts to data. • Understand how to compute probabilities of simple events, probabilities of compound events, and conditional probabilities. • Know how to make informed decisions using probabilities and expected values. • Understand how to use simulations to construct experimental probability distributions and to make information inferences about theoretical probability distributions. • Understand how to find probabilities involving finite sample spaces and independent trials.	• Mastery Exercises • Module Exam

6 – Discrete Math	• Analyze varied representations of sequences and series. • Apply recursive functions to model various mathematical phenomena. • Understand the principles of equivalence relations. • Contrast the multiple representations of discrete and continuous data. • Understand basic terminology and symbols of logic to real-world situations. • Apply counting techniques to solve real-world situations. • Interpret and illustrate basic set theory.	• Mastery Exercise • Module Exam
7 – Calculus: Limits & Continuity	• Understand the meaning of a limit of a function and how to calculate limits of functions, determine when the limit does not exist, and solve problems using the properties of limits. • Understand how to show that a particular function is continuous. • Understand the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem). • Know how to determine the limits of sequences, if they exist. • Become familiar with simple infinite series.	• Mastery Exercise • Module Exam
8 – Calculus: Derivatives	• Understand the derivative of a function as a limit, as the slope of a line tangent into a curve, and as a rate of change. • Know the relationship between continuity and differentiability. • Understand how to approximate derivatives and integrals numerically. • Understand how and when to use standard differentiation and integration techniques. • Understand how to analyze the behavior of a function (e.g., extrema, concavity, symmetry). • Understand how to apply derivatives to solve problems (e.g., related rates, optimization). • Understand the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem).	• Mastery Exercise • Module Exam
9 - Analyzing Functions Using Derivatives	• Justify conclusions about functions by applying the Mean Value Theorem over an interval. • Justify conclusions about functions by applying the Extreme Value Theorem. • Justify conclusions about the behavior of a function based on the behavior of its derivatives. • Calculate minimum and maximum values in applied contexts or analysis of functions. • Interpret minimum and maximum values calculated in applied contexts. • Determine critical points of implicit relations. • Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivatives.	• Mastery Exercise • Module Exam

10 – Calculus: Integration	● Understand how to approximate derivatives and integrals numerically. ● Understand how and when to use standard differentiation and integration techniques. ● Understand the foundational theorems of calculus (e.g., fundamental theorems of calculus, mean value theorem, intermediate value theorem). ● Understand integration as a limit of Riemann sums. ● Understand how to use integration to compute area, volume, distance or other accumulation processes.	● Mastery Exercise ● Module Exam
11 - Calculator Lab	● Practice using the on-screen graphing calculator that will be allowed during the Praxis exam.	● Lab
12 – Reflection and Final Exam	● Zoom meeting with instructor, if desired	● Reflection Paper ● Final Exam